The opinion in support of the decision being entered today was not written for publication and is not binding precedent of the Board.

UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE BOARD OF PATENT APPEALS
AND INTERFERENCES

Ex parte GEORGE EARL PETERSON

Appeal No. 2005-2760
Application 09/915,963

ON BRIEF

Before THOMAS, KRASS, and MACDONALD, Administrative Patent Judges.

KRASS, Administrative Patent Judge.

DECISION ON APPEAL

This is a decision on appeal from the final rejection of claims 1-3, 5-13, 15-19, 21, and 23-25.

The invention pertains to antenna structures. In particular, the inventive antenna structure comprises a tapered

antenna element coupled with a symmetrically shaped finite ground plane which supports the relatively wider directivity of the broadband structure. In another embodiment, the antenna structure is said to support a phase velocity greater than the speed of light.

Representative claims 1 and 2 are reproduced as follows:

1. An antenna structure comprising:

at least one antenna element, the at least one antenna element having at least one taper; and

a symmetrical finite ground plane coupled with the at least one antenna element.

2. The antenna structure of claim 1, wherein the at least one antenna element comprises a traveling wave antenna supporting a phase velocity greater than the speed of light.

The examiner relies on the following references:

Ogot	5,648,787	Jul. 15, 1997
Wicks	US H2016 H	Apr. 2, 2002 (Filed Mar. 5, 1986)

Claims 2 and 12 stand rejected under 35 U.S.C. § 112, first paragraph, as relying on a nonenabling disclosure.

Claims 1, 3, 5-9, 11, 13, and 15-18 stand rejected under 35 U.S.C. § 102 (e) as anticipated by Wicks.

Claims 10, 19, 21, and 23-25 stand rejected under 35 U.S.C. § 103 as unpatentable over Wicks in view of Ogot.

Reference is made to the briefs and answer for the respective positions of appellant and the examiner.

OPINION

Turning, first, to the rejection of claims 2 and 12 under 35 U.S.C. § 112, first paragraph, the examiner contends that the phrase, "the phase velocity being greater than the speed of light" "defies conventional theory of physics" (answer-page 3).

If the examiner had a reasonable basis for questioning the sufficiency of the disclosure, it was incumbent on appellant to come forward with evidence, if they could, to rebut the examiner's position. In re Buchner, 929 F.2d 660, 661, 18 USPQ2d 1331, 1332 (Fed. Cir. 1991).

As a matter of Patent and Trademark Office practice, a specification disclosure which contains a teaching of the manner and process of making and using the invention in terms which correspond in scope to those used in describing and defining the

subject matter sought to be patented *must* be taken as in compliance with the enabling requirement of the first paragraph of 35 U.S.C. § 112 *unless* there is reason to doubt the objective truth of the statements contained therein which must be relied on for enabling support. Assuming that sufficient reason for such doubt does exist, a rejection for failure to teach how to make and/or use will be proper on that basis; such a rejection can be overcome by suitable proofs indicating that the teaching contained in the specification is truly enabling, In re Marzocchi, 439 F.2d 220, 223, 169 USPQ 367, 369 (CCPA 1971); In re Sichert, 566 F.2d 1154, 1161, 196 USPQ 209, 215 (CCPA 1977).

When a rejection is made on the basis that the disclosure lacks enablement, it is incumbent upon the examiner to explain why he/she doubts the truth or accuracy of any statement in a supporting disclosure and to back up assertions with acceptable evidence or reasoning which is inconsistent with the contested statement.

Apparently, the examiner is taking the position that nothing can travel faster than the speed of light, as far as conventional physics is concerned, and that, therefore, any recitation of a

phase velocity being "greater than the speed of light" cannot be describing an enabling invention.

The trouble with the examiner's reasoning is that the examiner has not specifically identified exactly what "conventional theory of physics" is being referenced. As appellants argue, at page 5 of the principal brief, while there may be some notion that the speed of light is the upper bound on the speed at which things travel through space, this does not apply to basic physics principles as they relate to the phase velocity of an electromagnetic wave.

In particular, appellants cite a website, www.mathpages.com, specifically identifying the "Phase, Group, and Signal Velocity" portion thereof, indented under "Physics." Copies of pages 1-6 of that section were attached to appellants' response of September 10, 2003, and we attach same to this decision. At page 2 thereof, after defining "phase velocity" of a wave, the reference goes on to say that "there is no upper limit on the possible phase velocity of a wave," with an explanation as to how a general wave need not embody the causal flow of any physical effects. While a mere citation of a website is usually not probative because there is no assurance, as in, for example, a

published work, that the subject matter therein has been reviewed by legitimate authorities on the subject, the cited website, with its seemingly reasonable explanations, appears to offer some evidence tending to show the correctness of appellants' position. Moreover, the examiner's response, see infra, to appellants' argument appears to agree that a "fast wave" is a traveling wave having a velocity greater than the speed of light. Thus, the cited claim recitation does not defy the "conventional theory of physics," by the examiner's own admission.

It appears to us that appellants have provided a reasonable explanation and evidence to doubt the examiner's general statement of a phase velocity "greater than the speed of light" somehow defying a conventional theory of physics. The examiner has not advanced any evidence or an acceptable line of reasoning inconsistent with enablement, in view of the evidence submitted by appellant and, therefore, has not sustained his burden.

The examiner responds to appellant's evidence, at pages 6-7 of the answer, by arguing whether waves are "fast" or "slow" and whether the plane wave is in "free space" or not. The examiner then concludes by stating that claims 2 and 12 "need to meet two criteria one is **the traveling wave is the fast wave, and the**

other is in free space. None of applicant's invention meets these two criteria."

The examiner's explanation is not persuasive of nonenablement. The examiner now appears to be requiring appellant to add limitations into claims 2 and 12. Not only is the addition of limitations appellant's call, but, as appellant explains, at page 2 of the reply brief, the examiner's "requirement" is unnecessary since, by definition, a traveling wave having a velocity greater than the speed of light is already a fast wave in free space.

Since the examiner has not reasonably shown that having a phase velocity "greater than the speed of light," as claimed, would cause the skilled artisan to not be able to make and use the claimed invention, we will not sustain the rejection of claims 2 and 12 under 35 U.S.C. § 112, first paragraph.

Turning, now, to the rejection of claims 1, 3, 5-9, 11, 13, and 15-18 under 35 U.S.C. § 102(e), we also will not sustain this rejection.

It is the examiner's position that Wicks discloses, in Figures 1-5, the antenna structure claimed.

Appellant argues that Wicks lacks a teaching of the claimed "symmetrical finite ground plane." In particular, appellant points out that Wicks depicts a one-dimensional ground plane as a horizontal line and that this is a "typical depiction of an *infinite* ground plane" (principal brief-page 8). Appellant also points out that Figure 4 of Wicks shows a ground plane depicted in three-dimensions as an irregular plate, with the cut-away view "again suggesting an *infinite* ground plane" (principal brief-page 8). Appellant argues that Wicks gives no indication whatsoever that the ground planes depicted therein are "symmetrical" in any way.

The examiner's only response to appellant's allegations is that in Figure 5 of Wicks, the ground plane is shown as a finite ground plane, "the other figures depicting this ground plane are showing it in abbreviated form for convenience only. Second, the ground plane extends to infinity, this makes the ground plane symmetrical since extending to infinity is a form of translational symmetry" (answer-page 8).

While appellant presents no specific definition of "symmetrical finite ground plane," the examiner does not explain why the ground plane in Wicks is considered to be such a ground plane. The burden of proof is on the examiner in the first instance. In the instant case, the examiner has clearly not carried that burden in establishing anticipation of the instant claimed subject matter. It is not enough to say that a ground plane that extends to infinity must be a symmetrical finite ground plane, as claimed, without the examiner offering any definition of his/her own for the claimed term.

Since Wicks is entirely silent as to the matter of a symmetrical finite ground plane, we would need to resort to speculation to make any determination that Wicks, in fact, discloses such a ground plane. Deficiencies in the factual basis for an examiner's rejection cannot be supplied by resorting to speculation or unsupported generalities. In re Freed, 425 F.2d 785, 787, 165 USPQ 570, 571 (CCPA 1970); In re Warner, 379 F.2d 1011, 1017, 154 USPQ 173, 178 (CCPA 1967).

Accordingly, we will not sustain the rejection of claims 1, 3, 5-9, 11, 13, and 15-18 under 35 U.S.C. § 102(e).

However, we will sustain the rejection of claims 10, 19, 21, and 23-25 under 35 U.S.C. § 103.

Ogot is applied by the examiner for a teaching of a symmetrical disk shaped finite ground plane (elements 210, 250 in Figure 3A), alleged to be missing from Wicks. The examiner concluded that it would have been obvious to substitute the symmetrical disk shaped finite ground plane of Ogot for the metal ground plane of Wicks "in order to maximize the surface area of the ground plane perpendicular to the transmission element, and provides (sic) a uniform transmission pattern" (answer-page 6), referring to column 4, lines 66-67, and column 5, lines 1-3, of Ogot.

We note that appellant does not dispute the teachings of Ogot, but merely argues that the rejection is improper because the references "teach away" from each other since the artisan "would not be motivated to substitute the Ogot narrow band circular disk ground plane for the Wicks broadband ground plane" (principal brief-pages 11-12).

At the outset, we note that appellant has not denied that Ogot discloses a "symmetrical finite ground plane" that is "disk

shaped." Thus, the only issue here is whether the artisan would have combined the teachings of the two applied references.

The examiner has provided a rational basis for such a combination in citing Ogot's teaching that the employment of such a disk shaped finite ground plane has the advantage of maximizing the surface area of the ground plane perpendicular to the transmission element, and providing a uniform transmission pattern (column 5, lines 1-3, of Ogot), leading the artisan to use such a ground plane in Wicks.

We do not find persuasive appellant's argument that the references "teach away" from each other. It is appellant's position that Wicks describes a broadband antenna "which works best with a relatively large ground plane" and that Wicks' ground plane is much larger than the antenna elements. Appellant contrasts this with Ogot's teaching of a radar antenna in which the diameter of a circular ground plane is between $\lambda/8$ and $\lambda/4$, referring to column 3, lines 20-23, column 4, lines 61-64, and column 5, lines 11-21. Therefore, appellant concludes, at page 11 of the principal brief, once the diameter of Ogot's ground plane is set to satisfy one wavelength, it cannot simultaneously

satisfy the same requirement for a wide range of wavelengths demanded by the Wicks antenna.

Appellant's argument appears to presuppose that the artisan would make a direct substitution, or a bodily incorporation, of Ogot's ground plane for Wicks' ground plane. Clearly, the test of obviousness is not whether features of a secondary reference may be bodily incorporated into the primary reference's structure, nor whether the claimed invention is expressly suggested in any one or all of references; rather, the test is what the combined teachings of the references would have suggested to those of ordinary skill in the art. It is not necessary that a device shown in one reference can be physically inserted into the device shown in another reference to justify combining their teachings in support of a rejection. In re Keller, 642 F.2d 413, 425, 208 USPQ 871, 881 (CCPA 1981).

Wicks lacks a teaching of a symmetrical disk shaped finite ground plane, though the reference teaches an antenna structure having a ground plane. Ogot is alleged by the examiner to teach the symmetrical disk shaped finite ground plane, an allegation which has not been denied by appellant, and Ogot also provides a teaching of advantages attained by using such a symmetrical disk

shaped finite ground plane (column 5, lines 1-3). Accordingly, it would appear reasonable that the skilled artisan would have been led to employ such a disk shaped ground plane in other antenna structures, seeking the advantages taught by Ogot. Now, in applying such a teaching, the artisan would not, willy nilly, merely make a direct substitution but, rather, the artisan would have employed prudent engineering considerations. That is, contrary to appellant's implications in the "teaching away" argument, supra, it is clear that the artisan would have adjusted for the bandwidth size of the necessary ground plane. Merely because the "size" of the ground planes may be different in Wicks and Ogot, this does not, per se, indicate a "teaching away" since the artisan would have been expected to make adjustments in size, and other prudent engineering considerations, in adapting different antenna characteristics to differing environments.

Ogot's teaching of being able to maximize the surface area of the ground plane perpendicular to the transmission element, and to provide a uniform transmission pattern, by the use of a symmetrically disk shaped finite ground plane, in our view, would have clearly suggested to the artisan to use a ground plane having those characteristics in other antenna structures, such as in Wicks, in order to achieve similar advantages. We find no

deterrence to employing Ogot's teaching to Wicks because of Wicks' broadband antenna "which works best with a relatively large ground plane," as argued by appellant at page 11 of the principal brief.

Accordingly, we will sustain the rejection of claims 10, 19, 21, and 23-25 under 35 U.S.C. § 103.

We also note that, in our view, Ogot provides for the deficiencies of Wicks noted supra with regard to our reversal of the rejection of claims 1, 3, 5-9, 11, 13, and 15-18 under 35 U.S.C. § 102 (e). However, there is no rejection of these claims under 35 U.S.C. § 103 before us.

Accordingly, we make the following new ground of rejection under 37 CFR § 41.50(b):

Claims 1 and 11 are rejected under 35 U.S.C. § 103 as unpatentable over Wicks in view of Ogot for the reasons supra, anent the rejection of claims 10 and 19 under 35 U.S.C. § 103. Ogot clearly provides for the deficiencies of Wicks with regard

to the "symmetrical finite ground plane" deemed to be missing from Wicks in the rejection of claims 1 and 11 under 35 U.S.C. § 102(e).

We make the new ground of rejection against claims 1 and 11 because the limitations of these claims are clearly included in dependent claims 10 and 19, the rejection under 35 U.S.C. § 103 of which we sustained. Thus, claims 1 and 11 should be included in the rejection under 35 U.S.C. § 103 based on the Wicks/Ogot combination.

We make no representations or new grounds of rejection regarding claims 3, 5-9, 13 and 15-18. We leave those claims for the examiner to revisit if the examiner deems it advisable to make any findings regarding those claims and the application of the Wicks/Ogot combination thereto.

Since we have not sustained the rejection of claims 2 and 12 under 35 U.S.C. § 112, first paragraph, and the rejection of claims 1, 3, 5-9, 11, 13, and 15-18 under 35 U.S.C. § 102 (e), but we have sustained the rejection of claims 10, 19, 21, and 23-25 under 35 U.S.C. § 103, the examiner's decision is

affirmed-in-part. We also enter a new ground of rejection against claims 1 and 11, in accordance with 37 CFR § 41.50(b).

This decision contains a new ground of rejection pursuant to 37 CFR § 41.50(b) (effective September 13, 2004, 69 Fed. Reg. 49960 (August 12, 2004), 1286 Off. Gaz. Pat. Office 21 (September 7, 2004)). 37 CFR § 41.50(b) provides "[a] new ground of rejection pursuant to this paragraph shall not be considered final for judicial review."

37 CFR § 41.50(b) also provides that the appellant, <u>WITHIN TWO MONTHS FROM THE DATE OF THE DECISION</u>, must exercise one of the following two options with respect to the new ground of rejection to avoid termination of the appeal as to the rejected claims:

> (1) *Reopen prosecution.* Submit an appropriate amendment of the claims so rejected or new evidence relating to the claims so rejected, or both, and have the matter reconsidered by the examiner, in which event the proceeding will be remanded to the examiner. . . .
>
> (2) *Request rehearing.* Request that the proceeding be reheard under § 41.52 by the Board upon the same record. . . .

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No time period for taking any subsequent action in connection with this appeal may be extended under 37 CFR § 1.136(a)(1)(iv).

AFFIRMED-IN-PART
37 CFR § 41.50(b)

JAMES D. THOMAS)
Administrative Patent Judge)
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) BOARD OF PATENT
ERROL A. KRASS) APPEALS AND
Administrative Patent Judge) INTERFERENCES
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ALLEN R. MACDONALD)
Administrative Patent Judge)

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Appeal No. 2005-2760
Application 09/915,963

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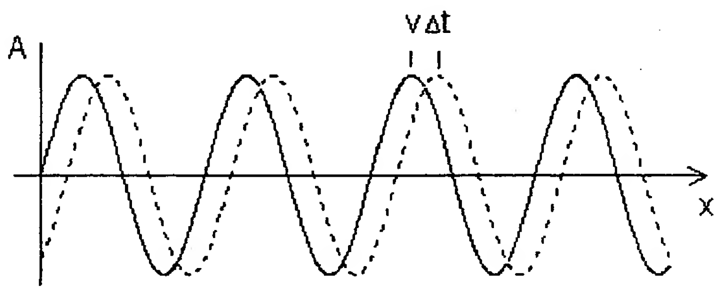
Phase, Group, and Signal Velocity

The velocity of a wave can be defined in many different ways, partly because there many different kinds of waves, and partly because we can focus on different aspects or components of any given wave. The ambiguity in the definition of "wave velocity" often leads to confusion, and we frequently read stories about experiments purporting to demonstrate "superluminal" propagation of electromagnetic waves (for example). Invariably, after looking into the details of these experiments, we find the claims of "superluminal communication" are simply due to a failure to recognize the differences between phase, group, and signal velocities.

In the simple case of a pure traveling sinusoidal wave we can imagine a "rigid" profile being physically moved in the positive x direction with speed v as illustrated below.



Clearly the wave function depends on both time and position. At any fixed instant of time, the function varies sinusoidally along the x axis, whereas at any fixed location on the x axis the function varies sinusoidally with time. One complete cycle of the wave can be associated with an "angular" displacement of 2π radians. The angular frequency ω of a wave is the number of radians per unit time at a fixed position, whereas the wave number k is the number of radians per unit distance at a fixed time. (If we prefer to speak in terms of cycles rather than radians, we can use the wavelength $\lambda = 2\pi/k$ and the frequency $\nu = \omega/2\pi$.) In terms of these parameters we can express a pure traveling wave as the function

$$A(t,x) = A_0 \cos(kx - \omega t)$$

where the "amplitude" A_0 is the maximum of the function. (We use the cosine function rather than the sine merely for convenience, the difference being only a matter of phase.) The minus sign denotes the fact that if we hold t constant and increase x we are moving "to the right" along the function, whereas if we focus on a fixed spatial location and allow time to increase, we are effectively moving "to the left" along the function (or rather, it is moving to the right and we are stationary). Reversing the sign gives $A_0 \cos(kx + \omega t)$, which is the equation of a wave propagating in the negative x direction. Note that the function A(t,x) is the fundamental solution of the (one-dimensional) "wave equation"

$$\frac{\partial^2 A}{\partial x^2} - \left(\frac{k}{\omega}\right)^2 \frac{\partial^2 A}{\partial t^2} = 0$$

Since ω is the number of radians of the wave that pass a given location per unit time, and 1/k is the spatial length of the wave per radian, it follows that $\omega/k = v$ is the speed at which the shape of the wave is moving, i.e., the speed at which any fixed phase of the

cycle is displaced. Consequently this is called the *phase velocity* of the wave, denoted by v_p. In terms of the cyclical frequency and wavelength we have $v_p = \lambda\nu$.

If we imagine the wave profile as a solid rigid entity sliding to the right, then obviously the phase velocity is the ordinary speed with which the actual physical parts are moving. However, we could also imagine the quantity "A" as the position along a transverse space axis, and a sequence of tiny massive particles along the x axis, each oscillating vertically in accord with $A_0 \cos(kx - wt)$. In this case the wave pattern propagates to the right with phase velocity v_p, just as before, and yet no material particle has any lateral motion at all. This illustrates that the phase of a traveling wave form may or may not correspond to a particular physical entity. It's entirely possible for a wave to "precess" through a sequence of material entities, none of which is moving in the direction of the wave. In a sense this is similar to the phenomenon of aliasing in signal processing. What we perceive as a coherent wave may in fact be simply a sequence of causally disjoint processes (like the individual spring-mass systems) that happen to be aligned spatially and temporally, either by chance or design, so that their combined behavior exhibits a wavelike pattern, even though there is no actual propagation of energy or information along the sequence.

Since a general wave (or wavelike phenomenon) need not embody the causal flow of any physical effects, there is obviously there is no upper limit on the possible phase velocity of a wave. However, even for a "genuine" physical wave, i.e., a chain of sequentially dependent events, the phase-velocity does not necessarily correspond to the speed at which energy or information is propagating. This is partly a semantical issue, because in order to actually convey information, a signal cannot be a simple periodic wave, so we must consider non-periodic signals, making the notion of "phase" somewhat ambiguous. If the wave profile never exactly repeats itself, then arguably the "period" of the signal must be the entire signal. On this basis we might say that the velocity of the signal is unambiguously equal to the "phase velocity", but in this context the phase velocity could only be defined as the speed of the leading (or trailing) edge of the overall signal.

In practice and common usage, though, we tend to define the "phase" of a signal with respect to the intervals between consecutive local maxima (or minima, or zero crossings). To illustrate, consider a signal consisting of two superimposed sine waves with slightly different frequencies and wavelengths, i.e., a signal with the amplitude function

$$A(x,t) \;=\; \cos\big[(k-\Delta k)x-(\omega-\Delta\omega)t\big] \;+\; \cos\big[(k+\Delta k)x-(\omega+\Delta\omega)t\big]$$

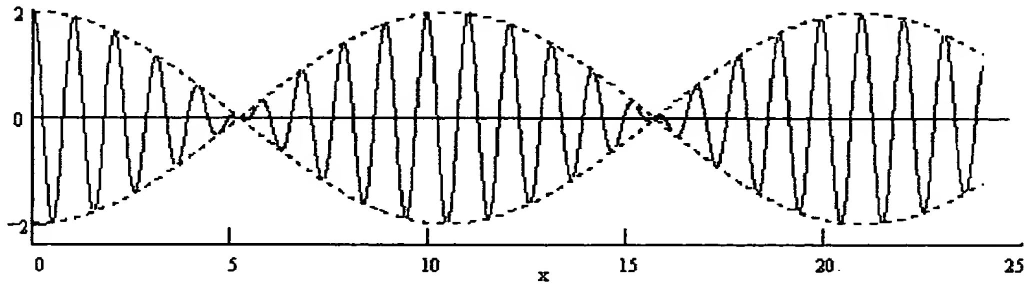
As most people know from experience, the combination of two slightly unequal tones produces a "beat", resulting from the tones cycling in and out of phase with each other. Using a well-known trigonometric identity we can express the two components of this signal in the form

$$\cos\big[(kx-\omega t)\pm(\Delta kx-\Delta\omega t)\big]$$
$$=\; \cos(kx-\omega t)\cos(\Delta kx-\Delta\omega t) \;\mp\; \sin(kx-\omega t)\sin(\Delta kx-\Delta\omega t)$$

Therefore, adding the two terms of A(x,t) together, the products of sines cancel out, and we can express the overall signal as

$$A(x,t) \;=\; 2\cos(kx-\omega t)\cos(\Delta kx-\Delta\omega t)$$

This can be somewhat loosely interpreted as a simple sinusoidal wave with the angular velocity ω, the wave number k, and the modulated amplitude $2\cos(\Delta kx - \Delta\omega t)$. In other words, the amplitude of the wave is itself a wave, and the phase velocity of this modulation wave is $v = \Delta\omega/\Delta k$. A typical plot of such a signal is shown below for the case $\omega = 6$ rad/sec, $k = 6$ rad/meter, $\Delta\omega = 0.1$ rad/sec, $\Delta k = 0.3$ rad/meter.



The "phase velocity" of the internal oscillations is $\omega/k = 1$ meter/sec, whereas the amplitude envelope wave (indicated by the dotted lines) has a phase velocity of $\Delta\omega/\Delta k = 0.33$ meter/sec. As a result, if we were riding along with the envelope, we would observe the internal oscillations moving forward from one group to the next.

The propagation of information or energy in a wave always occurs as a change in the wave. The most obvious example is changing the wave from being absent to being present, which propagates at the speed of the leading edge of a wave train. More generally, some modulation of the frequency and/or amplitude of a wave is required in order to convey information, and it is this modulation that represents the signal content. Hence the actual speed of content in the situation described above is $\Delta w/\Delta k$. This is the phase velocity of the amplitude wave, but since each amplitude wave contains a group of internal waves, this speed is usually called the *group velocity*.

Physical waves of a given type in a given medium generally exhibit a characteristic group velocity as well as a characteristic phase velocity. This is because within a given medium there is a fixed relationship between the wave number and the frequency of waves. For example, in a transparent optical medium the refractive index n is defined as the ratio c/v_p where c is the speed of light in vacuum and v_p is the phase velocity of light in that medium. Now, since $v_p = \omega/k$, we have $\omega = kc/n$. Bearing in mind that the refractive index is typically a function of the frequency (resulting in the "dispersion" of colors seen in prisms, rainbows, etc), we can take the derivative of ω as follows

$$\frac{d\omega}{dk} = \frac{c}{n} - \frac{ck}{n^2}\frac{dn}{dk}$$

Hence any modulation of an electromagnetic wave in this medium will propagate at the group velocity

$$v_g = v_p\left[1 - \frac{k}{n}\frac{dn}{dk}\right]$$

In a medium whose refractive index is constant, independent of frequency (such as the

vacuum), we have dn/dk = 0 and therefore the group velocity equals the phase velocity. On the other hand, in most commonly observable transparent media (such as air, water, glass, etc.) at optical frequencies have a refractive indices that increase slightly as a function of wave number and (therefore) frequency. This is why the high frequency (blue) components of a beam of white light are deflected more than the low frequency (red) components as they pass through a glass prism. It follows that the group velocity of light in such media (called dispersive) is less than the phase velocity.

It is quite possible for the phase velocity of a perfectly monochromatic wave of light, assuming such a thing exists, to exceed the value of c, because it conveys no information. In fact, the concept of a perfectly monochromatic beam of light is similar to the idea of a "free photon", in the sense that neither of them has any physical significance or content, because a photon must be emitted and absorbed, just as a beam of light cannot be infinite in extent and duration, but must always have a beginning and an end, which introduces a range of spectral components to the signal. Any modulation will propagate at the group velocity, which, in dispersive media, is always less than c.

An example of an actual physical application in which we must be careful to distinguish between the phase and the group velocity is the case of electromagnetic waves propagating through a hollow magnetic conductor, often called a *waveguide*. A waveguide imposes a "cutoff frequency" ω_0 on any propagating electromagnetic waves based on the geometry of the tube, and will not sustain waves of any lower frequency. This is roughly analogous to how the pipes in a Church organ will sustain only certain resonant patterns. As a result, the dominant wave pattern of a propagating wave with a frequency of ω will have a wave number k given by

$$k = \frac{1}{c}\sqrt{\omega^2 - \omega_0^2}$$

Since (as we've seen) the phase velocity is ω/k, this implies that the (dominant) phase velocity in a waveguide with cutoff frequency ω_0 is

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{\omega_0}{\omega}\right)^2}}$$

Hence, not only is the phase velocity generally greater than c, it approaches infinity as ω approaches the cutoff frequency ω_0. However, the speed at which information and energy actually propagates down a waveguide is the group velocity, which (as we've seen) is given by $d\omega/dk$. Taking the derivative of the preceding expression for k with respect to w gives

$$\frac{dk}{d\omega} = \frac{\omega}{c\sqrt{\omega^2 - \omega_0^2}}$$

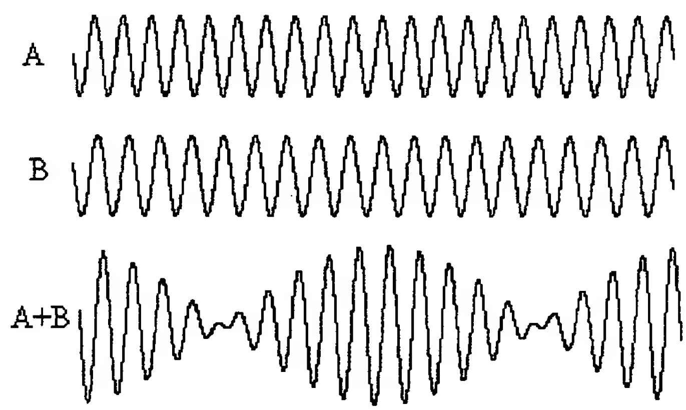
so the group velocity in a waveguide with cutoff frequency ω_0 is

$$v_g = \frac{d\omega}{dk} = c\sqrt{1 - \left(\frac{\omega_0}{\omega}\right)^2}$$

which of course is always less than or equal to c.

Unfortunately we frequently read in the newspapers about how someone has succeeded in transmitting a wave with a *group* velocity exceeding c, and we are asked to regard this as an astounding discovery, overturning the principles of relativity, etc. The problem with these stories is that the group velocity corresponds to the actual signal velocity only under conditions of normal dispersion, or, more generally, under conditions when the group velocity is less than the phase velocity. In other circumstances, the group velocity does not necessarily represent the actual propagation speed of any information or energy. For example, in a regime of anomalous dispersion, which means the refractive index *decreases* with increasing wave number, the preceding formula shows that what we called the group velocity exceeds what we called the phase velocity. In such circumstances the group velocity no longer represents the speed at which information or energy propagates.

To see why the group velocity need not correspond to the speed of information in a wave, notice that in general, by superimposing simple waves with different frequencies and wavelengths, we can easily produce a waveform with a group velocity that is arbitrarily great, even though the propagation speeds of the constituent waves are all low. A snapshot of such a case is shown below. In this figure the sinusoidal wave denoted as "A" has a wave number of $k_A = 2$ rad/meter and an angular frequency of $\omega_A = 2$ rad/sec, so it's individual phase velocity is $v_A = 1$ meter/sec. The sinusoidal wave denoted as "B" has a wave number of $k_B = 2.2$ rad/meter and an angular frequency of $\omega_B = 8$ rad/sec, so it's individual phase velocity is $v_B = 3.63$ meters/sec.



The sum of these two signals is denoted as "A+B" and, according to the formulas given above, it follows that this sum can be expressed in the form $2\cos(kx-\omega t)\cos(\Delta kx-\Delta\omega t)$ where $k = 5$, $\omega = 2.1$, $\Delta k = 0.1$, and $\Delta\omega = 3$. Consequently, the "envelope wave" represented by the second factor has a phase velocity of 30 meters/sec. Nevertheless, it's clear that no information can be propagating faster than the phase speeds of the constituent waves A and B. Indeed if we follow the midpoint of a "group" of A+B as it proceeds from left to right, we find that when it reaches the right hand side it consists of the sum of peaks of A and B that entered at the left long before the current "group" had

even "appeared". This is just one illustration of how simple interfering phase effects can be mis-construed as ultra-high-speed signals. In fact, by simply setting k_A to 2.2 and k_B to 2.0, we can cause the "groups" of A+B to precess from *right to left*, which might mistakenly be construed as a signal propagating *backwards in time*!

Needless to say, we have *not* succeeded in arranging for a message to be received before it was sent, nor even in transmitting a message superluminally. Examples of this kind merely illustrate that the "group velocity" does not always represent the speed at which real information (or energy) is moving. This stands to reason, because the two cosine factors of the carrier/modulation waveform are formally identical, so we can't arbitrarily declare one of them to represent the carrier and the other to represent the modulation. Both are required, so we shouldn't expect the information speed to be any greater than the lesser of the two phase speeds, nor should it exceed the lesser of the phase speeds of the two components A and B. Furthermore, we already know that the transmission of information via an individual wave such as A will propagate at the speed of an incremental disturbance of A, which depends on how dw and dk are related to each other. In the example above we arbitrarily selected increments of ω and k, but our ability to do this in a physical context would depend on a great deal of flexibility in the wave propagation properties of the medium. This is where the ingenuity of the experimenter can be deployed to arrange various exotic substances and fields in such a way as to permit the propagation of waveforms with the desired properties. The important point to keep in mind is that none of these experiments actually achieves meaningful superluminal transfer of information.

Incidentally, since we can contrive to make the "groups" propagate in either direction, it's not surprising that we can also make them stationary. Two identical waves propagating in opposite directions at the same speed are given by

$$A_0 \cos(kx \pm \omega t) = A_0\left[\cos(kx)\cos(\omega t) \mp \sin(kx)\sin(\omega t)\right]$$

Superimposing these two waves propagating (with synchronized nodes) in opposite directions yields a standing pure wave

$$A_0 \cos(kx \pm \omega t) + A_0 \cos(kx \pm \omega t) = 2A_0 \cos(kx)\cos(\omega t)$$

Another common source of confusion regarding the propagation speed of periodic physical effects is the phenomenon of "numerator dynamics", discussed in the note on Lead-Lag Frequence Response.

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